Enclosure of the Range of a Complex Polynomial Over a Complex Interval

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Bounding the range of a function over a given region is an important task which is inherent in a remarkable variety of problems in mathematics and many of its applications. These include quantitative estimation of the remainder terms in numerical integration and differentiation, sensitivity analysis of systems, the certification of properties of function like monotonicity, convexity, and univalence, and branch and bound methods in global optimization, to name only a few. In this talk, we consider complex polynomials which arise in many areas such as control systems, image and signal processing, coding theory, and electrical networks. The regions over which the range of such polynomials are sought are axis-aligned compact regions in the complex plane called complex intervals. The tool we are using is the expansion of the given polynomial into Bernstein polynomials, see, e.g., [3]. The convex hull of the coefficients of this expansion, the so-called Bernstein coefficients, provides an enclosure for the range of the given polynomial over the complex interval. In contrast to the case of real polynomials, the use of the Bernstein polynomials for finding an enclosure for the range of a complex polynomial over a region in the complex plane has been considered in only a few papers so far. The first paper in this regard was [3] in which the range of a complex polynomial over the interval $[0, 1]$ is enclosed. Rokne [4] and Grassmann and Rokne [1] extended this result to range enclosures over complex intervals and discs in the complex plane. Furthermore, they considered enclosures for the range of complex polynomials with coefficients which are not exactly known but can located within complex intervals or discs. Alternative methods to the Bernstein expansion include the so-called circular complex forms [5] and [2, Chapter 2].

We first briefly recall the expansion of a multivariate real polynomial into Bernstein polynomials over a box and some of its fundamental properties as well as from [7], see also [6], a matrix method for the computation of the Bernstein coefficients. We present the Bernstein expansion for a complex polynomial which is applied for finding an upper bound for the modulus of a polynomial. It turns out that the computation of the range of a complex polynomial over a complex interval can be reduced to the calculation of the range over its boundary. We discuss some methods for the computation of the Bernstein coefficients of a complex polynomial and extend them to multivariate complex polynomials and to rational functions as well as to complex interval polynomials. Furthermore, we show how the Bernstein coefficients of a degree elevated expansion can be calculated from those of the real part of the lower degree expansion. As a byproduct, we obtain new matrix methods for the multivariate real Bernstein expansion, viz. the computation of the Bernstein coefficients of a product of two polynomials and the computation of the Bernstein coefficients of a polynomial from those of its partial derivatives.

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