# A Tuning Rule Based on Internal Model Control and the Nyquist Criterion

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*Abstract*— **If the process contains a delay (dead time), the Nyquist criterion is well suited to derive a PI or PID tuning rule because the delay is taken into account without approximation. The tuning of the speed of the closed loop enters naturally by the crossover frequency. The goal of robustness and performance is translated into the phase margin.**

## I. Introduction

The notion of specifying the speed of a control loop during the design procedure of a controller is quite common in advanced control technology. If, for instance, the design is based on the frequency response of the process, the speed selection enters into the design procedure in terms of bandwidth or crossover frequency. If the design is based on a transfer function or a state-space model of the process, the speed selection comes into the game very directly if pole-placement is realized [17]. If the design of an observer and a state-space controller is carried out via algebraic Riccati equations, the speed is selected indirectly by turning up and down the weighting matrices **Q** and **R** and the covariance matrices **W** and **V** [8][10].

In the wide field of process control, there are plants with about hundred control loops. In short time, with simple experiments and without provoking process hazards, the technicians and engineers have to tune the feedback controllers. A frequency response is often not available because of experimental difficulties and limited time. A dynamic process model is completely out of reach, the layout calculations consider only the stationary flows, pressures, temperatures, velocities etc. Therefore, controllers are often tuned based on the step response of the process. The old tuning rules of Ziegler and Nichols [16], Chien, Hrones and Reswick [3] and Cohen and Coon [4] are still very popular [12], but they do not offer any possibility to specify the speed of the control loop. Their formulas for the PI / PID controller parameters depend only on two or three process parameters determined from the step response. If the process reacts fast, the closed control loop is made fast as well. This, however, is a significant drawback, because unnecessary control action leads to higher energy consumption, a shorter lifetime of the actuator and a bad influence on coupled control loops.

A tuning rule should allow – or better – demand to specify the desired speed of disturbance rejection as a tuning knob to meet the requirements of the application. This desirable property is fulfilled by the development of so-called IMC tuning rules, starting from Dahlin [5] via several stages of improvement [11], [13], [14] until Sgogestad's improved rule SIMC+[15], [6]. The user has to specify a closed-loop time constant $\tau_c$. The theoretical foundation is pole placement involving an approximation regarding the delay of the process.

This article shows a similar approach, also starting from a simple process model (internal model) but reaching the control design via the Nyquist criterion. So the delay is exactly taken into account. The user has to specify the crossover frequency (or bandwidth) as tuning knob for the speed of disturbance rejection.

## II. Prerequisites and Definitions

The main purpose of the control loop (Fig. 1) is compensating the process disturbance by the adequate counter-action of the actuator. This view is often called the *regulator problem*. Åström and Hägglund [1] emphasize, that "most PID controllers operate as regulators", that the performance with respect to process disturbance ("load disturbance") is the main goal.

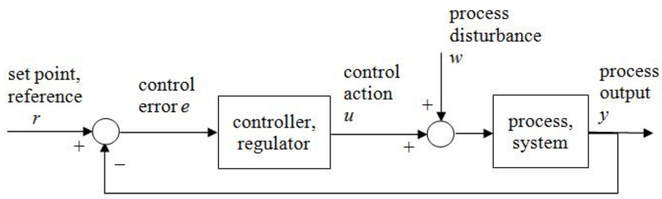


Figure 1. Block diagram of the closed control loop with process disturbance at process input, so-called load disturbance.

The process is known only by a measurement of the step response. The graphical evaluation works with the tangent at the point of the steepest slope, compare IEC 60050-351 [9], Fig. 4, or [7]. For proportional systems the resulting three system parameters are

1. the *delay time* $T_u$, also known as *equivalent dead time*,
2. the *balancing time* $T_g$, and
3. the *stationary gain* $k_S$.

For integrating systems, the balancing time never ends ($T_g = \infty$). The resulting two system parameters are

1. the *delay time* $T_u$
2. the *integrating gain* or *velocity gain* $k_v$ [15], [2].

The controller structures used are either PI

$$G_{R,PI}(s) = k_P \cdot \left(1 + \frac{1}{T_i \cdot s}\right) \tag{1}$$

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or PIDT1 ((2), so-called *ideal controller in series with a first-order lag* [11]), often called PID for reasons of shortness:

$$G_{R,PID}(s) = k_P \cdot \frac{1 + T_d \cdot s + \dfrac{1}{T_i \cdot s}}{1 + T_1 \cdot s} \ . \qquad (2)$$

## III. Theoretical Background Based on the Nyquist Stability Criterion

Given the process parameters $T_u$, $T_g$, $k_S$ or $T_u$, $k_v$, respectively, for integrating systems, the model of the process is not characterized. There are several more or less soft, more or less oscillating process step responses possible, yielding the same parameters $T_u$, $T_g$, $k_S$. The distinction between mainly first-order dynamics (showing a corner at the end of the delay) and mainly second-order dynamics (showing a soft increase) is necessary if it comes to the question whether derivative control (see section *B*) is possible [2], [15]. For the design of a PI controller, derived in section *A*, the issue of order determination is not crucial. The internal model is based on the worst-case assumption of first-order dynamics.

### *A. PI control*

A common assumption in the field of IMC-based tuning rules [5], [11], [13], [14], [15], [6] is modelling the process as first-order lag with time delay FOTD (or FOLPD [11]) with the transfer function

$$G_S(s) = k_S \cdot \left( \frac{1}{1 + T_g \cdot s} \right) \cdot \exp(-T_u \cdot s) \qquad (3)$$

Equation (3) includes lag-dominated and delay-dominated processes. The lag delay ratio $r = T_g/T_u$ was introduced by Chien, Hrones and Reswick [3]. Equivalently, the normalized time delay $\tau = T_u/(T_u + T_g) = 1/(1+r)$ is used by Åström and Hägglund [2] eq. (2.19).

The aims of the PI tuning rule are the specified crossover frequency and a good damping, i.e. a maximum sensitivity $M_s \le 2$. For the following calculation this aim is replaced by a phase margin of $\varphi_m$=60° for simplicity. The open-loop transfer function (from (1), (3)) is

$$G_O(s) = k_P \cdot \left( 1 + \frac{1}{T_i \cdot s} \right) \cdot k_S \cdot \left( \frac{1}{1 + T_g \cdot s} \right) \cdot \exp(-T_u \cdot s) \qquad (4)$$

resulting in an open-loop phase curve

$$\angle G_O(i\omega) = -\arctan\left( \frac{1}{T_i \cdot \omega} \right) - \arctan(T_g \cdot \omega) - T_u \cdot \omega \qquad (5)$$

A phase margin of 60° fixes the open-loop phase curve at the crossover circle frequency $\omega_c$ to

$$\angle G_O(i\omega_c) = -120° = -\frac{2\pi}{3} \qquad (6)$$

resulting in a design equation for $T_i$ depending on $\omega_c$

$$-\frac{2\pi}{3} = -\arctan\left( \frac{1}{T_i \cdot \omega_c} \right) - \arctan(T_g \cdot \omega_c) - T_u \cdot \omega_c \qquad (7)$$

However, the crossover frequency cannot be selected without limitation; otherwise (7) has no solution or yields an inadequately weak integral action. In order to develop a simple tuning rule, the maximum crossover frequency $f_{cm}$ is chosen so that the last term in (7) makes up for –30° or –π/6

$$2\pi \cdot f_{cm} = \omega_{cm} = \frac{1}{T_u} \cdot \frac{\pi}{6} \qquad (8)$$

This formula for the maximum speed of PI control has the additional advantage to result in the popular concept [11] eq. (5.44) called *compensation of the system time constant*

$$T_i = T_g \qquad (9)$$

If the crossover frequency is intentionally tuned slower, if $f_c \le f_{cm}$ the design equation (7) is more relaxed. The additional phase angle of some degrees may be used to increase the integral action.

$$T_i = \frac{1}{\omega_c \cdot \tan\left( \dfrac{2\pi}{3} - \arctan(T_g \cdot \omega_c) - T_u \cdot \omega_c \right)} \qquad (10)$$

A formula like in (10) seems too complicated for the purpose of an easily applicable tuning rule. It is simplified further for the important case of a long balancing time $T_g$, because otherwise, if $T_g$ is short, the setting of (10) does not differ much from (9) which is a good choice for short $T_g$, anyway. In the limit $T_g \to \infty$, (10) simplifies to

$$T_i = \frac{1}{\omega_c \cdot \tan\left( \dfrac{\pi}{6} - T_u \cdot \omega_c \right)} \qquad (11)$$

which is further simplified by approximating the tangent for small angles by the angle itself.

$$T_i = \frac{1}{\omega_c \cdot \left( \dfrac{\pi}{6} - T_u \cdot \omega_c \right)} = \frac{3}{\pi^2} \cdot \frac{1}{f_c \cdot (1 - 12 \cdot T_u \cdot f_c)} \qquad (12)$$

Putting all the arguments together, the minimum of the both candidates, calculated by (11) and (12) is selected, to find the maximum integral action.

However, there are extreme cases (e.g. $T_g \approx 0$ or $f_c = f_{cm}$ along with $T_g = \infty$) which would still result in extreme values for the integral action time constant $T_i$ .

In order to bind $T_i$ to the crossover frequency in a sensible way, it is restricted to the interval

$$T_i \in \left[ \frac{0.1}{\omega_c}, \frac{5}{\omega_c} \right] \qquad (13)$$

It makes no sense to reduce $T_i$ below $0.1/\omega_c$ because the controller is dominated by integral action, anyway. On the other hand, it makes no sense to increase $T_i$ over $5/\omega_c$ because the weak integral action makes the final approach of $y$ to $r$ very slow. These thoughts go along with the improvements of the IMC tuning by Skogestad. First [14], he limits $T_i \le 4\cdot(\tau_c + T_u)$ in the SIMC rule. Then [15], he introduces an additional low limit $T_i \ge T_u/3$ in the improved SIMC+ rule.

Practical experience has shown that the reduction of the phase margin by the setting $T_i \cdot \omega_c = 5$ does not have a bad influence on the closed-loop damping. Why –arctan(1/5)= –0.197rad = –11.3° only slightly affects the maximum sensitivity $M_S$ , can be understood by focusing on the distance of the open-loop curve $G_o(i\omega)$ to the critical point –1 in the Nyquist plot

$$s_m = \min_{\omega} |1 - G_o(i\omega)| = \frac{1}{M_S} \quad . \tag{14}$$

This distance is hardly affected because –arctan(1/( $T_i \cdot \omega$)) decreases rapidly for $\omega > \omega_c$ inside the unit sircle. Compare the controller setting with $f_c = f_{cm}$ =0.833 Hz in the Table I. Although the phase margin is less than 60°, the sensitivity is still well below 2.

The proportional gain of the PI controller is finally calculated from the Nyquist equation defining the crossover frequency

$$|G_o(i\omega_c)| = 1 \tag{15}$$

$$|G_{R,PI}(i\omega_c)\cdot G_S(i\omega_c)| = 1 \tag{16}$$

$$k_P \cdot \sqrt{1 + \left(\frac{1}{T_i\cdot\omega_c}\right)^2} \cdot k_S \cdot \frac{1}{\sqrt{1+(T_g\cdot\omega_c)^2}} = 1 \tag{17}$$

$$k_P = \frac{T_i\cdot\omega_c}{k_S}\cdot\sqrt{\frac{1+(T_g\cdot\omega_c)^2}{1+(T_i\cdot\omega_c)^2}} \tag{18}$$

*B. PID control*

For mainly second-order system dynamics, a faster control action than achieved by a PI controller (8) is possible. The derivative action of PIDT1 control lifts the open-loop phase curve. This opens the door to a higher crossover frequency. Around this crossover frequency, the open-loop amplitude diagram decays with a slope –1, because the derivative action (+1) is combined with the second-order system (–2):

$$|G_O(i\omega)| = |G_R(i\omega)|\cdot|G_S(i\omega)| \sim \omega^1 \cdot \omega^{-2} = \omega^{-1} \tag{19}$$

The following equations for the PID setting are based on further goals regarding simplicity and robustness:

- Dependence on only two process parameters: $T_u$ and the quotient $k_S/T_g$=$k_v$. For the fast dynamics addressed now, the slow part of the process dynamics does not matter.
- No explicit dependence on the desired crossover frequency. If $f_{cm}$ is not fast enough, a speed-up factor of 3 is not too high. Faster PID setting is not possible with the limited knowledge about the process.
- Maximum sensitivity $M_S \le 2$, for all those processes with very soft rise of the step response.
- Maximum sensitivity inside the unit circle. This is very important to assure, that gain reduction leads to less sensitivity, better damping, better robustness.

The internal model for the development of the PIDT1 controller tuning rule is

$$G_S(s) = \frac{k_v}{s}\cdot\frac{1}{1+T_{Lag}\cdot s}\cdot\exp\left(-(T_u - T_{Lag})\cdot s\right) \tag{20}$$

The lag time constant $T_{Lag}$ may vary between 0 (pure first-order dynamics with delay) and $T_u$ (pure second-order dynamics). In the following, a nominal value of $T_{Lag} = 0.8\cdot T_u$ is assumed. The variation is addressed later.

Further inserting the fixed crossover frequency

$$f_c = \frac{1}{4\cdot T_u} \tag{21}$$

which means three times faster than with PI control (8) yields the phase angle of the process

$$\angle G_S(i\omega_c) = -0.6\pi - \arctan(0.4\pi) = -2.784 \tag{22}$$

A phase margin of at least 60°

$$\angle G_R(i\omega_c) + \angle G_S(i\omega_c) \ge -\frac{2\pi}{3} = -2.094 \tag{23}$$

can only be reached by a phase lift of the controller

$$\angle G_R(i\omega_c) \ge 0.690 \tag{24}$$

$$\arctan\left(T_d\cdot\omega_c - \frac{1}{T_i\cdot\omega_c}\right) - \arctan(T_1\cdot\omega_c) \ge 0.690 \tag{25}$$

This is one design equation for the three controller time constants $T_i$, $T_d$, $T_1$. A strong derivative action is parametrized by the author’s choice

$$T_i\cdot\omega_c = 2.827 \quad T_d\cdot\omega_c = 2.356 \quad T_1\cdot\omega_c = 0.157 \tag{26}$$

$$\angle G_R(i\omega_c) = 0.952 \tag{27}$$

establishing a larger phase margin in order to be robust against the variation of $T_{Lag}$.

The proportional gain of the PIDT1 controller is finally calculated from the Nyquist equation (15) about the crossover frequency

$$k_P = 1.15\cdot\frac{1}{k_v\cdot T_u} \tag{28}$$

*C. Verification of PID control*

The open-loop transfer function $G_O(s)$ does not depend on $k_v$. The frequency is scalable by $1/T_u$. So the only remaining degree of freedom is the ratio of $T_{Lag}/T_u$ (20). It is varied in three steps: 0.6, 0.8, 1.0. With $T_{Lag}$=$0.6\cdot T_u$ the system has a significant delay, the rise of the step response is not as soft as that of a second-order system. $T_{Lag}$=$0.8\cdot T_u$ stands for the nominal internal model. With $T_{Lag}$=$1.0\cdot T_u$ the system has a perfect second-order acceleration without delay. Fig. 2 shows the Bode diagrams of the system, the controller and the combined open loop. For the nominal case (solid), the design goals regarding speed (21) and phase margin 75° (27) are confirmed. The variation $T_{Lag}$=1.0·$T_u$ (dashed) slightly reduces the speed and increases the phase margin. This is not a problem. However, the variation $T_{Lag}$=0.6·$T_u$ (dash-dotted) increases the speed and reduces the phase margin to 47°.

Fig. 3 displays the same results in the complex plane of the Nyquist curve. The nominal case (solid) fulfills the design goal regarding the maximum sensitivity with $M_S$ =1.59, the variation $T_{Lag}$=1.0·$T_u$ (dashed) even better with $M_S$ =1.10. However, with the variation $T_{Lag}$=0.6·$T_u$ (dash-dotted) the maximum sensitivity $M_S$ =3.2 is inacceptable. In this situation, the user of the tuning rule should detune [2] the controller by gain-reduction. Reducing the proportional gain to 70% restores reasonable robustness and performance with $M_S$ =1.97. It is obvious from Fig. 3 that the success of this detuning procedure relies on the fact that the closest approach to the critical point lies inside the unit circle.

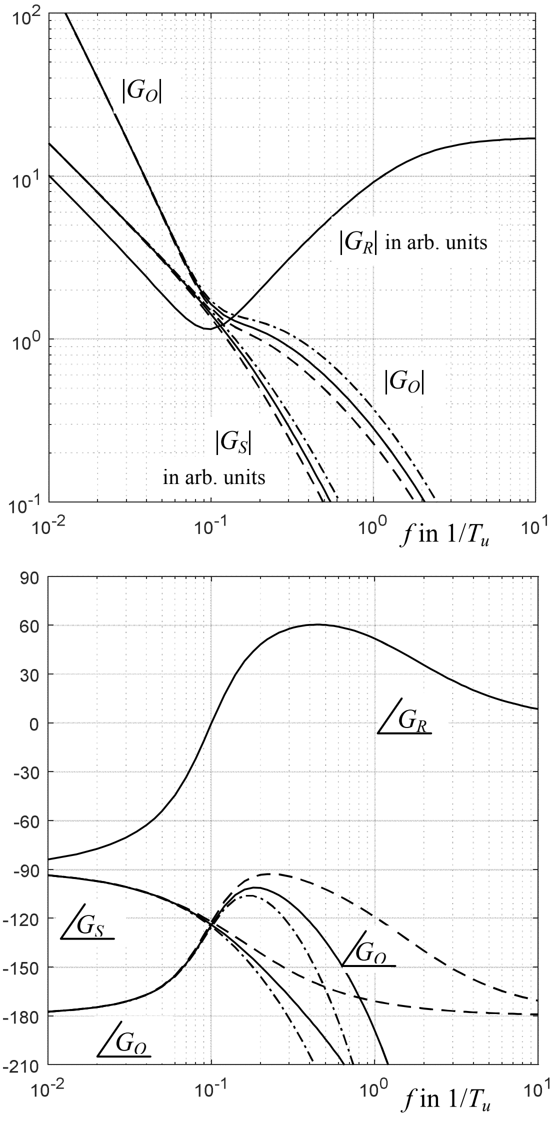


Figure 2. Bode diagrams of the system $G_S$, the controller $G_R$ and the open loop $G_O$. The case of the nominal system $T_{Lag}/T_u$=0.8 is shown as solid line. The variations of $T_{Lag}/T_u$=0.6 (dash-dotted) and 1.0 (dashed) have considerable influence on the crossover frequency and the phase margin.

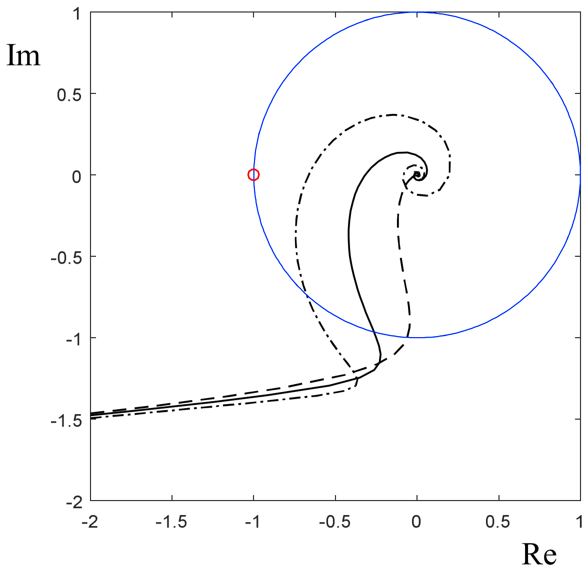


Figure 3. Nyquist plots of the open loop. The case of the nominal system $T_{Lag}/T_u$=0.8 is shown as solid line. The variations of $T_{Lag}/T_u$=0.6 (dash-dotted) and 1.0 (dashed) strongly influence the distance to the critical point, i.e. the maximum sensitivity.

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## IV. Essential Equations for Users of the Tuning Rule

The developed tuning rule is named REGULECT (regulator with selectable speed). It guides the user to the decision whether PI or PIDT1 control is appropriate.

After evaluating the process step response the user specifies the desired crossover frequency $f_{cd}$. Now, the tuning rule is applied in the following steps.

### A. The maximum PI crossover frequency

The delay time sets a limit to the speed of PI control.

$$f_{cm} = \frac{1}{12 \cdot T_u} \quad (29)$$

### B. The decision about the controller structure

If PI control is fast enough, i.e. if $f_{cd} \le f_{cm}$ , PI-control ((31) to (33)) is selected with $f_c = f_{cd}$ , as desired.

Otherwise PID control is considered. If the process dynamics is mainly of second-order type, which may be judged from the soft increase of the slope of the step response, AND the sensor signal is not noisy, fast PID control ((33), (34)) is selected with $f_c = 1/(4 \cdot T_u)$.

If at least one of the two prerequisites for differential control is not fulfilled, that means if the step-response shows a corner (first-order system) or the sensor signal is noisy, PI control ((30) to (32)) is selected with reduced crossover frequency $f_c = f_{cm}$.

*C. The controller parameters*

The setting of the **PI controller** with $f_c$ and $\omega_c = 2\pi \cdot f_c$:

$$T_i = \max\left(\min\left(T_g, \frac{3}{\pi^2} \cdot \frac{1}{f_c \cdot (1 - 12 \cdot T_u \cdot f_c)}, \frac{5}{\omega_c}\right), \frac{0.1}{\omega_c}\right) \quad (30)$$

$$k_P = \frac{T_i \cdot \omega_c}{k_S} \cdot \sqrt{\frac{1 + (T_g \cdot \omega_c)^2}{1 + (T_i \cdot \omega_c)^2}} \quad (31)$$

In case of an integrating system the limit $T_g \to \infty$ and the replacement of $T_g/k_S$ by $1/k_v$ result in (32):

$$k_P = \frac{T_i \cdot \omega_c^2}{k_v \cdot \sqrt{1 + (T_i \cdot \omega_c)^2}} \quad (32)$$

The setting of the **PIDT1 controller**:

$$T_i = 1.8 \cdot T_u \qquad T_d = 1.5 \cdot T_u \qquad T_1 = 0.1 \cdot T_u \quad (33)$$

$$k_P = 1.15 \cdot \frac{T_g}{k_S \cdot T_u} \quad (34)$$

The limit value for integrating systems makes (34) result in (28).

*D. The closed-loop test*

The closed-loop performance is experimentally tested. If a slower or faster speed should be preferred, the design is repeated with a revised setting of the tuning knob, the desired crossover frequency.

In the special case of a PIDT1 controller and a badly damped closed-loop performance (compare dash-dotted lines in Figs. 2 and 3) the origin of the problem lies in the difficulty of order determination from the step response (compare [2][15]). The user may either switch to the PI controller with reduced crossover frequency $f_c = f_{cm}$ or keep the PIDT1 structure and chose gain-reduction.

## V. Application to a Practical Example

The dynamic process is presented in Fig. 4. The water mass flow $y$ is to be controlled through a control valve with stroke $u$.

The process disturbance $w$ may be manually introduced by a ball valve mounted in the tube before the control valve. Slightly reducing the cross-section implies a negative disturbance.

The user specifies a crossover frequency of $f_{cd}$ =0.23 Hz. The REGULECT tuning rule guides to a PI controller with $T_i$ = 0.169s and $k_P$ =7.12 (mm·s)/kg. The desired speed is realized: $f_c = f_{cd}$ . The closed-loop experiment (Fig. 5) shows a precise control action, as fast as desired. Oscillations (about 3Hz and 50Hz) which are faster than $f_c$ are nicely ignored by the controller. This bears a great benefit: The energy consumption of the compressed air and the wear of the valve are reduced, resulting in low life-cycle costs of the plant.

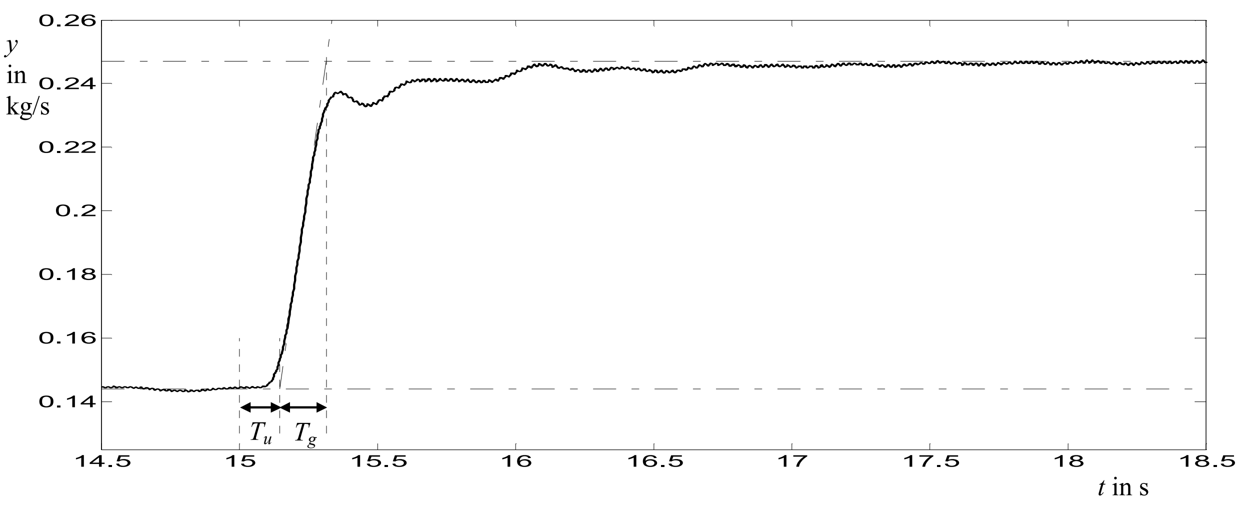


Figure 4. Measured step response of mass flow to a 3mm greater opening of the control valve. Evaluation of the tangents yields the delay time $T_u$ =0.147s, the balancing time $T_g$ =0.169s and the proportional gain $k_S$ = 0.0343 kg/(s·mm) .

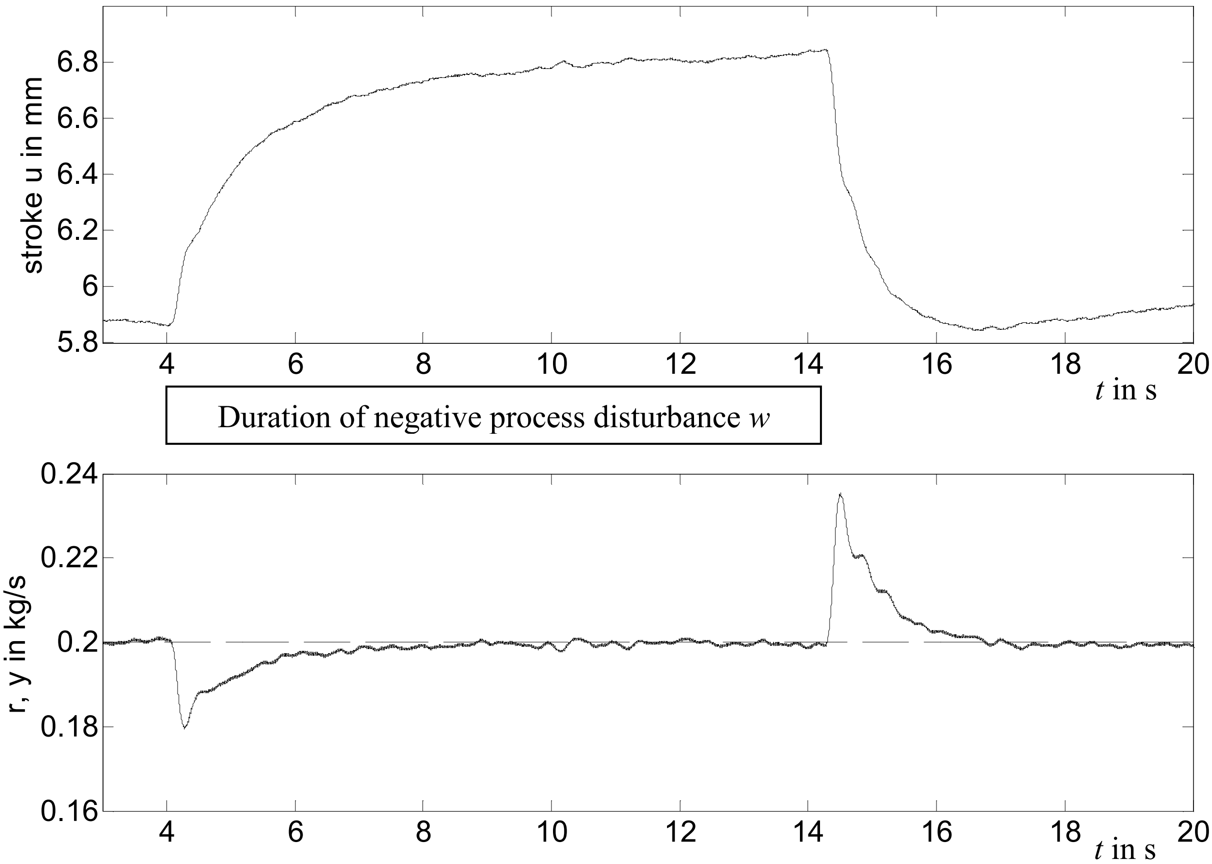


Figure 5. Closed-loop response to a disturbance, beginning with a negative step at $t$=4s (partial closure of the ball valve) and ending at $t$=14.3s with a positive step (opening of the ball valve).

## VI. APPLICATION TO A THEORETICAL EXAMPLE

This example, a third-order integrating system plus time delay, is chosen differently from the assumed internal models (3), (20) to prove the robustness of the tuning rule.

$$G_S(s) = \frac{k_v}{s} \cdot \frac{1}{(1+T_A \cdot s)\cdot(1+T_B \cdot s)} \cdot \exp(-T_t \cdot s) \qquad (35)$$

The parameters $k_v$ =1.0 $s^{-1}$, $T_A$ = 0.075 s, $T_B$ = 0.01 s, $T_t$ = 0.015 s give $T_u$= 0.1 s.

The REGULECT tuning rule is applied with three different settings of the desired crossover frequency $f_{cd}$= 0.1 Hz, 0.5 Hz, 2 Hz. Since the maximum PI crossover frequency is $f_{cm}$=0.833 Hz (29), the two slower settings are realized by PI controllers and the faster setting is realized by a PIDT1 controller. Equation (21) states, that the crossover frequency with the PIDT1-controller will be 2.5 Hz, slightly faster than desired.

Fig. 6 shows simulation results. The process disturbance $w$ excites the closed loop by a step function (regulator problem). The process output $y$ reacts, and as a consequence the control action $u$ rises until finally compensating the process disturbance. So, the process output $y$ returns to the set point ($r$=0).

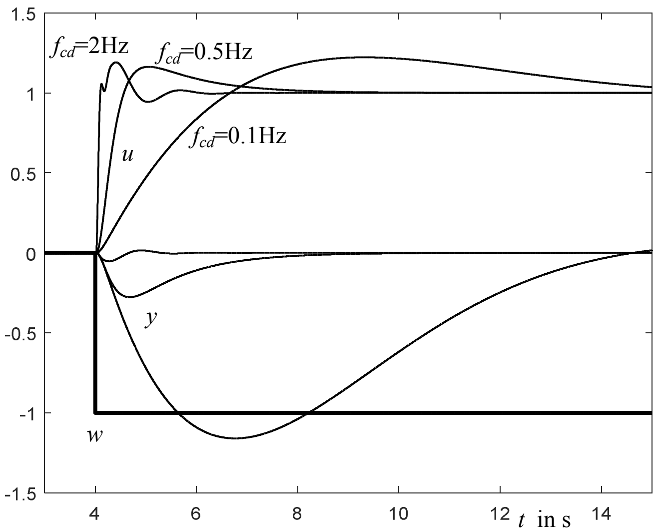


Figure 6. Closed-loop response to a disturbance step at $t$=4s ($w$, thick line) with the system (35) and three different controllers, designed by REGULECT with desired crossover frequencies $f_{cd}$= 0.1 Hz, 0.5 Hz, 2 Hz.

TABLE I. FOUR REGULECT CONTROLLERS OF DIFFERENT SPEED FOR THE SYSTEM (35). THE THIRD LINE, THE FASTEST POSSIBLE PI CONTROLLER, IS ADDITIONALLY INSERTED TO SUPPORT THE ARGUMENTS IN SECTION III.A.

| designed | | | | | | realized | | |
|---|---|---|---|---|---|---|---|---|
| controller | $k_P$ | $T_I$ s | $T_D$ s | $T_1$ s | $f_c$ Hz | $f_c$ Hz | $\varphi_m$ | $M_S$ |
| PI | 0.571 | 3.45 | 0 | 0 | 0.100 | 0.100 | 62° | 1.06 |
| PI | 3.075 | 1.52 | 0 | 0 | 0.500 | 0.488 | 61° | 1.27 |
| *PI* | *5.132* | *0.96* | *0* | *0* | *0.833* | *0.783* | *51°* | *1.48* |
| PIDT1 | 11.500 | 0.18 | 0.15 | 0.010 | 2.500 | 2.658 | 71° | 1.70 |

Figure 6 and Table I show that the tuning of the speed of disturbance rejection works successfully. The REGULECT tuning rule guides to the appropriate controller structure PI or PIDT1, respectively. The three controllers (slow PI, medium PI, fast PIDT1) lead to convincing curves of the control action $u$ and the system output $y$. The designed speed (6th column in Table I) is nearly exactly realized (7th column) in the closed loop although the system differs from the internal models (3), (20) used in the theoretical derivation in section III.

## VII. CONCLUSION AND OUTLOOK

This article develops a PI/PID tuning rule on the Nyquist criterion as foundation. The rule, named REGULECT, allows selecting the speed of the control loop in terms of the crossover frequency. The equations for applying the rule are summarized in section IV. They are easily applicable and offer a transparent guidance to the decision, if a PI controller or a PID controller is to be used.

Future research should facilitate the order determination from the measured step response. For instance a graphical method for the quantitative determination of the lag time constant $T_{Lag}$ in (20) would be very worthwhile. The tuning rule equations would become more precise if $T_{Lag}$ could be incorporated. The variation shown in Figs. 2 and 3 would be covered by the improved equations of the future tuning rule.

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